Schrödinger's Sparsity in the Cross Section of Stock Returns

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Sparsity

A central challenge in modern statistics: **addressing high-dimensional problems**

Sparse modeling and variable selections

- **selection** for **sparse** models: L_1 penalty
- *Researchers assume that the underlying signal is sparse.*

Empirical asset pricing:

- Feng, Giglio, and Xiu (JF 2020) and Freybergr, Neuhierl, and Weber (RFS 2020)
- *Assumption: the cross section is driven by a few factors/chars.*

Sparsity

A central challenge in modern statistics: **addressing high-dimensional problems**

Dense modeling and regularization

- **Shrinkage**: L_2 penalty

Empirical asset pricing:

- Kozak, Nagel, and Santosh (JFE 2020) and Kozak and Nagel (WP 2023): Char-sorted factors / IPCA type factors / Slope factors do not span the SDF unless a large number of chars are used simultaneously.
- Shen and Xiu (WP 2025): When signals are weak, Ridge outperforms Lasso for prediction. Equivalently, the predictive model might not be sparse.

Illusion of Sparsity

Giannone, Lenza, and Primiceri (ECTA 2021) (GLP2021) propose a Bayesian sparse model that parametrizes the level of sparsity

- Link L_1 and L_2: **no assumption**, but posterior.

- They examine various types of datasets (Macro / Finance / Micro)

- Findings: the posterior distribution does not typically concentrate on a single sparse model.

⇒ This phenomenon highlights an illusion of sparsity in economic data.

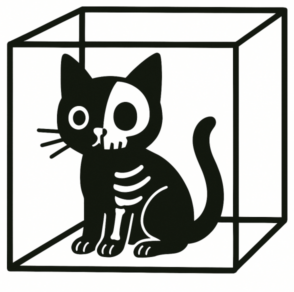
Illusion of Sparsity

- Statisticians have developed lots of tools:
  - **Shrinkage**: L_2 penalty.
  - **Variable selection**: L_1 penalty.
- AP modeling choices — Sparse v.s. Dense
- These modeling outcomes are often artifacts of the imposed prior.
- A less frequently explored question arises:

Are asset pricing models sparse?

Motivation: Schrödinger's Sparsity

Existing approaches: require researchers to commit *ex ante* to either a sparse (selection) or dense (shrinkage) specification before the empirical investigation and adhere to that assumption throughout the modeling process.



Schrödinger's cat

- We cannot determine whether the cat is alive or dead until we open the box.

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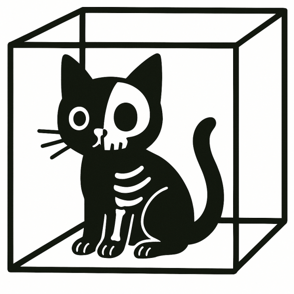

Schrödinger's cat

- We cannot determine whether the cat is alive or dead until we open the box.
- We cannot determine whether the model is **sparse or dense** until we **examine the data**.

Research Questions

We examine the sparsity of chars in AP models.

- We study the sparsity level following GLP2021.
- Our framework is built on the conditional latent factor model of IPCA (Kelly, Pruitt, and Su, JFE 2019) and the Bayesian latent factor model (Geweke and Zhou, RFS 1996).
- Our focuses are the char-driven alpha (mispricings) and beta (factor loadings).

Extend the class of conditional factor models in which alphas and betas depend on firm characteristics (e.g., Jagannathan and Wang, 1996 JF; Lettau and Ludvigson, 2001 JPE; Avramov, 2004 RFS; Kelly, Pruitt, and Su, JFE 2019; Bybee, Kelly, and Su, RFS 2023; Fan, Ke, Liao, Neuhierl, 2024 WP)

Respond to the ongoing debate over sparsity versus complexity in asset pricing (e.g., Kozak, Nagel, Santosh, 2020 JFE; Kozak and Nagel, WP 2023; He, Zhao, Zhou, 2024 WP; Kelly, Malamud, Zhou, 2024 JF; Shen and Xiu, 2025 WP)

Literature Positions

Advance the literature on Bayesian model selection, averaging, and shrinkage in finance (e.g., Avramov, 2002 JFE; Barillas and Shanken, 2018 JF; Chib, Zeng, and Zhao, 2020 JF; Chib, Zhao, and Zhou, 2024 MS; Avramov, Cheng, Metzker, Voigt, 2023 JF; Bryzgalova, Huang, Julliard, 2023 JF)

Complement new methodologies that extract latent factors from high-dimensional signals (e.g., Lettau and Pelger, 2020 RFS; Kim, Korajczyk, and Neuhierl 2021 RFS; Gu, Kelly, and Xiu, 2021 JoE; Chen, Pelger, Zhu, 2024 MS; Feng, He, Polson, Xu, 2024 JFQA; Cong, Feng, He, HE, 2025 JFE)

Our findings complement the literature on time-varying and regime-dependent models of expected returns and factor loadings (e.g., Ferson and Harvey, 1999 JF; Lewellen and Nagel, 2006 JFE; Smith and Timmermann, 2021 RFS)

Contribution

Methodology Innovations

We propose a novel Bayesian sparse conditional (latent) factor model.

- We allow sparsity levels to be freely estimated (or fixed exogenously).
- We can consider the (global or separate) sparsity of alphas and betas.
- Extension: our approach provides a new alternative to estimate conditional models of observable factors (plus latent factors).
  - For example, conditional CAPM
  - recover unspanned risk factors

Empirical Findings

- ...

Key Results

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- Sparsity varies across test asset sets.

  5×5 ME-BM portfolios ⇒ Sparse model

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- Best-performing models are neither extremely sparse nor dense.
- Alpha is typically sparser than beta.

  Complementary relationship: when betas are dense, alpha becomes more concentrated, and vice versa.
- Sparsity varies across test asset sets.

  5$\times$5 ME-BM portfolios $\Rightarrow$ Sparse model
- Sparsity is time-varying. Models become more sparse during recessions.

Model

Full Model

$$
\begin{aligned}
r_{i,t} &= \boldsymbol{\alpha}(\mathbf{Z}_{i,t-1}) + \boldsymbol{\beta}(\mathbf{Z}_{i,t-1})\mathbf{f}_t + \epsilon_{i,t} \qquad (1)\\
\text{where} \quad \boldsymbol{\alpha}(\mathbf{Z}_{i,t-1}) &= \alpha_0 + \boldsymbol{\alpha}_1 \mathbf{Z}_{i,t-1}\\
\boldsymbol{\beta}(\mathbf{Z}_{i,t-1}) &= \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1(\mathbf{I}_K \otimes \mathbf{Z}_{i,t-1})\\
\epsilon_{i,t} &\sim \mathcal{N}\left(0, \sigma_i^2\right)
\end{aligned}
$$

- $r_{i,t}$: return of asset i at time t
- $\mathbf{f}_t$: K latent factors
- $\mathbf{Z}_{i,t-1}$: vector, L chars for asset i at time $t-1$

Spike-and-Slab Prior

Spike-and-slab prior, a Bayesian variable selection prior.

$$P(\beta \neq 0) = q, \quad P(\beta = 0) = 1 - P(\beta \neq 0) = 1 - q.$$

$$\beta = \begin{cases} \mathcal{N}\left(0, \gamma^2\right) \text{ with prob } q & \text{The regressor is chosen. } \sim L_2 \text{ penalty} \\ 0 \text{ with prob } 1 - q & \text{The regressor is not chosen. } \sim L_1 \text{ penalty} \end{cases}$$

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- Standard spike-and-slab prior: q is a specific value.
- GLP2021: q has its prior so that one can sample q.
  - These priors probabilistically interpolate between variable selection and shrinkage, allowing the degree of sparsity to be estimated from the data.
- **Prior settings of $q \neq$ precise control of sparsity levels!**

Sparse BayesIPCA Model

$$r_{i,t} = \alpha_0 + \boldsymbol{\alpha}_1 \mathbf{Z}_{i,t-1} + \boldsymbol{\beta}_0 \mathbf{f}_t + \boldsymbol{\beta}_1 [\mathbf{f}_t \otimes \mathbf{Z}_{i,t-1}] + \epsilon_{i,t}.$$

- Independent spike-and-slab priors on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$
- **Global prior**: same sparsity level of alpha and beta

$$[\boldsymbol{\alpha}_1, \boldsymbol{\beta}_1] \overset{iid}{\sim} \begin{cases} \mathcal{N}\left(0, \gamma^2\right) & \text{with prob } q \\ 0 & \text{with prob } 1 - q \end{cases}$$

$$q \sim \text{Beta}(a_q, b_q),$$

$$\gamma^2 \sim \mathrm{IG}(A/2, B/2)$$

$$\alpha_0, \boldsymbol{\beta}_0 \overset{iid}{\sim} \mathcal{N}\left(0, \xi^2\right), \quad \xi^2 \sim \mathrm{IG}(C/2, D/2)$$

Sparse BayesIPCA Model

$$r_{i,t} = \alpha_0 + \boldsymbol{\alpha}_1 \mathbf{Z}_{i,t-1} + \boldsymbol{\beta}_0 \mathbf{f}_t + \boldsymbol{\beta}_1 [\mathbf{f}_t \otimes \mathbf{Z}_{i,t-1}] + \epsilon_{i,t}.$$

- Independent spike-and-slab priors on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$
- **Separate priors**: different sparsity levels of alpha and beta.

$$\boldsymbol{\alpha}_1 \overset{iid}{\sim} \begin{cases} \mathcal{N}\left(0, \gamma_\alpha^2\right) & \text{with prob } q_\alpha \\ 0 & \text{with prob } 1 - q_\alpha \end{cases}, \quad \boldsymbol{\beta}_1 \overset{iid}{\sim} \begin{cases} \mathcal{N}\left(0, \gamma_\beta^2\right) & \text{with prob } q_\beta \\ 0 & \text{with prob } 1 - q_\beta \end{cases}$$

$q_\alpha \sim \text{Beta}(a_{q_\alpha}, b_{q_\alpha})$, $q_\beta \sim \text{Beta}(a_{q_\beta}, b_{q_\beta})$,

$\gamma_\alpha^2 \sim \text{IG}(A_\alpha/2, B_\alpha/2)$, $\gamma_\beta^2 \sim \text{IG}(A_\beta/2, B_\beta/2)$,

Sparse BayesIPCA Model: An Extension

$$r_{i,t} = \alpha_0 + \boldsymbol{\alpha}_1 \mathbf{Z}_{i,t-1} + \boldsymbol{\beta}_0 \mathbf{f}_t + \boldsymbol{\beta}_1 [\mathbf{f}_t \otimes \mathbf{Z}_{i,t-1}] + \epsilon_{i,t}.$$

Directly control the sparsity level (i.e., control # selected chars).

M restricts the number of chars driving alpha and beta.

- **(Global) joint prior**:

$$(\tau_1, \tau_2, \cdots, \tau_L) \sim \prod_{i=1}^{L} \text{Bernoulli}(L) \times \mathbf{I}\left(\sum_{i=1}^{L} \tau_i = M\right)$$

- **(Separate) joint priors**:

$$(\tau_1^{\alpha}, \tau_2^{\alpha}, \cdots, \tau_L^{\alpha}) \sim \prod_{i=1}^{L} \text{Bernoulli}(L) \times \mathbf{I}\left(\sum_{i=1}^{L} \tau_i^{\alpha} = M_{\alpha}\right)$$

$$(\tau_1^{\beta_k}, \tau_2^{\beta_k}, \cdots, \tau_L^{\beta_k}) \sim \prod_{i=1}^{L} \text{Bernoulli}(L) \times \mathbf{I}\left(\sum_{i=1}^{L} \tau_i^{\beta} = M_{\beta}\right)$$

Empirical Results

Main test assets:

- P-Tree (Cong, Feng, He, and He, JFE 2025) test assets (1990-2024)
  - Constructed based on the past sample (1980-1989)

Other test assets:

- 25 ME/BM portfolios (FF25), 61 long-short portfolios for each characteristic (LS61), 357 bivariate-sorted portfolios (Bi357).
- 500 stocks with the highest and 500 stocks with the lowest average market equity (Big ind500 / Small ind500).

(i) Global and Separate Sparsity

Table 1: Model Performance under Global Sparse Priors

		CSR^2			Sharpe		
		$K=1$	$K=3$	$K=5$	$K=1$	$K=3$	$K=5$
Panel A: Unrestricted # selected chars.							
	0.1	29.37	**43.66**	**55.57**	0.35	1.36	0.92
q prior mean	0.5	29.54	43.63	54.79	0.35	1.44	0.92
	0.9	**29.71**	43.62	53.89	**0.35**	**1.50**	**0.95**
Panel B: Fixed # selected chars.							
	2	25.44	**52.49**	**51.02**	**0.44**	**1.11**	0.48
M	10	29.53	38.32	41.51	0.35	0.87	**1.12**
	18	27.48	39.31	42.02	0.33	0.55	0.95
Panel C: No sparsity							
M	20	**29.92**	36.88	45.23	0.35	0.57	0.95

Benchmark: CAPM.

CSR^2: model's ability to explain the cross-sectional expected return.

q prior mean is 0.1. $K = 5 \sim M_\alpha = 1, M_\beta = 9$.

(i) Global and Separate Sparsity

Table 2: Model Performance under Separate Sparse Priors on Alphas and Betas

		CSR^2			TP. Sp		
		$K=1$	$K=3$	$K=5$	$K=1$	$K=3$	$K=5$
Panel A: Unrestricted # selected chars.							
(q_α prior mean, q_β prior mean)	0.1,0.1	29.17	44.09	**59.20**	0.34	0.75	0.71
	0.5,0.1	29.37	43.27	58.47	0.35	0.77	0.79
	0.9,0.1	29.41	43.54	58.00	0.35	1.14	0.68
	0.1,0.5	29.29	43.53	57.82	0.34	0.75	1.00
	0.5,0.5	29.48	42.49	56.84	0.35	1.01	1.14
	0.9,0.5	29.53	43.65	54.94	0.35	1.17	0.92
	0.1,0.9	29.48	**45.11**	58.72	0.34	0.99	0.77
	0.5,0.9	29.64	42.48	56.84	0.35	1.00	**1.14**
	0.9,0.9	**29.73**	44.13	56.69	**0.35**	**1.27**	0.90
Panel B: Fixed # selected chars.							
(M_α, M_β)	2,2	25.44	49.34	48.39	**0.44**	**1.10**	0.95
	10,2	27.98	51.07	50.10	0.37	0.57	0.87
	18,2	25.17	47.01	38.00	0.32	0.79	0.68
	2,10	28.85	51.17	56.83	0.42	0.60	0.87
	10,10	29.59	37.87	41.20	0.35	0.89	0.97
	18,10	27.19	40.97	39.03	0.32	0.47	0.88
	2,18	29.81	**54.91**	**56.99**	0.43	0.65	1.13
	10,18	**29.88**	34.24	51.26	0.36	1.01	**1.22**
	18,18	27.46	39.30	42.11	0.33	0.53	0.94

(i) Global and Separate Sparsity

- **Unrestricted # selected chars**:
  - Global prior:

    q prior mean is 0.1. $K = 5 \sim M_\alpha = 1, M_\beta = 9$.
  - Separate priors:

    Both prior means of q_α and q_β are 0.1. $K = 5 \sim M_\alpha = 1, M_\beta = 10$.
- **Fix # selected chars**:
  - Global prior: $K = 5 \sim M_\alpha = 2, M_\beta = 2$
  - Separate priors: $K = 5 \sim M_\alpha = 2, M_\beta = 18$.

(i) Global and Separate Sparsity

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- **Fix # selected chars**:
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- Best-performing models are neither extremely sparse nor dense.
- # chars driving betas exceeds that of those driving alpha.
- When sparsity is imposed exogenously, model performance peaks when the imposed level aligns with the endogenous level chosen by the posterior.

(ii) Large Sets of Test Assets

Table 3: Sparsity for Different Test Assets

	Global prior			Separate priors			
	q	M_α	M_β	q_α	q_β	M_α	M_β
Panel A: P-Tree							
100	0.48	5	11	0.31	0.59	4	12
200	0.60	7	14	0.40	0.67	5	14
400	0.70	9	15	0.47	0.85	9	18
Panel B: Ind. Stock							
Small 500	0.62	11	13	0.51	0.65	9	13
Big 500	0.68	8	16	0.41	0.82	6	18
Panel C: Others							
FF25	0.41	1	10	0.20	0.50	1	10
LS61	0.67	4	17	0.24	0.83	2	17
Bi357	0.81	11	19	0.50	0.90	10	19

- Sparsity levels vary across different types of test assets.

  E.g., FF25 sparser.

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- Panel A: Within the same category of test assets, a larger number of assets generally requires more chars.

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- Panel B: Those test assets that are harder to explain tend to require more chars to capture alpha.

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- Panel B: Complementary relationship: when betas are dense, alpha becomes more concentrated, and vice versa.

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- Panel C: There is substantial variation in the sparsity levels across commonly used test assets.

(iii) Time-varying Sparsity

Table 4: Time Variation Analysis: Sparsity in Regimes

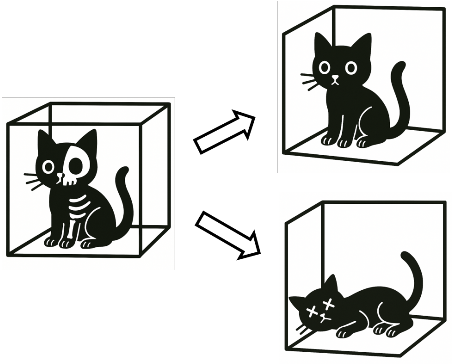
	Different periods					
	Regime1	Regime2	Regime3	Normal	Recession	Full
Panel A: Global prior						
q	0.37	0.41	0.42	0.47	0.42	0.48
Panel B: Separate priors						
q_α	0.30	0.29	0.23	0.27	0.24	0.31
q_β	0.42	0.46	0.56	0.54	0.53	0.59

- Settings of time periods:
  - Follow breakpoints in Smith and Timmermann (RFS 2021) to split time periods. (July 1998 and June 2010)
  - Define recession periods based on the Sahm Rule, totaling 88 months.
- AP models tend to be sparser during recessions.

Schrödinger's Sparsity

Sparsity levels vary across both **cross-sectional** and **time-series** dimensions.

$\Rightarrow$ i) Type and number of test assets; ii) Time periods / Macro conditions



Assuming AP model to be either sparse or dense ex ante may be wrong.

Conditional CAPM

Model with Observable and Latent Factors

- In the conditional observable factor model, alpha and beta can be (sparse) functions of high-dimensional chars.
- Augmenting latent factors helps recover unspanned risk factors in observable factor models.

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$$
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r_{i,t} &= \boldsymbol{\alpha}(\mathbf{Z}_{i,t-1}) + \boldsymbol{\beta}(\mathbf{Z}_{i,t-1}) \underbrace{\left[\tilde{\mathbf{f}}_t, \mathbf{f}_t\right]}_{\mathbf{F}_t} + \epsilon_{i,t} \\
&= \underbrace{\alpha_0 + \boldsymbol{\alpha}_1 \mathbf{Z}_{i,t-1}}_{\text{mispricing}} + \underbrace{\boldsymbol{\beta}_0 \tilde{\mathbf{f}}_t + \boldsymbol{\beta}_1 [\tilde{\mathbf{f}}_t \otimes \mathbf{Z}_{i,t-1}]}_{\text{obs. factors, conditional beta}} + \underbrace{\boldsymbol{\beta}_0 \mathbf{f}_t + \boldsymbol{\beta}_1 [\mathbf{f}_t \otimes \mathbf{z}_{i,t-1}]}_{\text{latent factors, dynamic loadings}} + \epsilon_{i,t}.
\end{aligned}
$$

Table 5: Augmented Observable Factor Models

	CSR^2	Sharpe	(q_α, q_β)	$\beta_{0,\text{MKT}}$	$\boldsymbol{\alpha}$ RMSE
Panel A: only obs					
MKT	14.93	0.57	0.45,0.63	1.15	0.0032
FF5	50.38	1.13	0.26,0.61	1.07	0.0014
Panel B: only latent					
LF1	29.48	0.35	0.49,0.53	/	0.0036
LF5	56.81	1.13	0.23,0.34	/	0.0011
Panel C: obs + latent					
MKT+LF1	53.87	0.87	0.31,0.65	1.14	0.0015
MKT+LF5	56.45	1.39	0.24,0.46	0.98	0.0007
FF5+LF1	50.55	1.23	0.33,0.65	1.06	0.0012
FF5+LF5	60.33	1.53	0.18,0.42	0.95	0.0001
Panel D: uncond. model					
MKT	/	0.57	/	1.19	0.0060
FF5	49.25	1.13	/	1.09	0.0042

- Panel A v.s. Panel C: Adding latent factors helps mitigate model misspecification.
  - $\beta_{0,\text{MKT}}$: be closed to 1 after introducing latent factors.
  - $\boldsymbol{\alpha}$ RMSE: decreases after introducing latent factors.

(iv) Resurrecting Conditional CAPM

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MKT+LF1	53.87	0.87	0.31,0.65	1.14	0.0015
MKT+LF5	56.45	1.39	0.24,0.46	0.98	0.0007
FF5+LF1	50.55	1.23	0.33,0.65	1.06	0.0012
FF5+LF5	60.33	1.53	0.18,0.42	0.95	0.0001
Panel D: uncond. model					
MKT	/	0.57	/	1.19	0.0060
FF5	49.25	1.13	/	1.09	0.0042

- Panel A v.s. Panel D: The conditional factor model outperforms in cross-sectional explanatory power.

(iv) Resurrecting Conditional CAPM

Figure 1: Chars Importance in Alphas and Betas across Different Models

	α_1	$\beta_{1,MKT}$
α_0 , $\beta_{0,MKT}$	−0.0014	1.1507
ABR	0.0000	−0.0411
ACC	−0.0002	0.0020
ADM	0.0000	−0.0090
AGR	−0.0000	0.0383
BASPREAD	−0.0020	−0.0002
BETA	−0.0016	0.1464
BM	0.0000	0.0359
CFP	0.0006	−0.0405
EP	0.0011	−0.0595
ME	0.0000	−0.0001
MOM1M	0.0016	−0.0159
MOM12M	0.0017	−0.0394
NI	0.0000	−0.0576
OP	0.0000	0.0007
RDM	0.0013	0.0005
ROE	0.0000	0.0003
SEAS1A	0.0000	0.0393
SP	0.0019	−0.0307
SUE	0.0018	−0.0274
SVAR	−0.0001	0.2309

Colour scale: 0, 0.2, 0.4, 0.6, 0.8, 1

(a) MKT

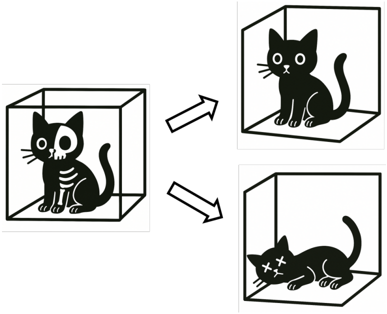
	α_1	β_{1,LF_1}	$\beta_{1,MKT}$
α_0 , β_{0,LF_1} , $\beta_{0,MKT}$	0.0000	−0.5604	1.1409
ABR	0.0000	−0.0022	−0.0156
ACC	−0.0000	−0.0546	0.0015
ADM	0.0003	0.0017	−0.0210
AGR	0.0000	0.0115	0.0384
BASPREAD	−0.0001	−0.1224	−0.0032
BETA	−0.0010	−0.0945	0.1278
BM	0.0001	0.1802	−0.0016
CFP	−0.0000	−0.0111	−0.0303
EP	0.0001	0.1403	−0.0448
ME	−0.0018	0.3740	−0.0136
MOM1M	0.0000	−0.0029	−0.0357
MOM12M	0.0014	0.0365	−0.0192
NI	0.0000	0.0429	−0.0305
OP	0.0001	0.2001	−0.0001
RDM	0.0000	0.0003	−0.0001
ROE	0.0000	−0.0003	0.0008
SEAS1A	−0.0000	0.0410	0.0255
SP	0.0001	0.2445	−0.0006
SUE	0.0014	0.1628	−0.0330
SVAR	−0.0003	−0.5751	0.2263

Colour scale: 0, 0.2, 0.4, 0.6, 0.8, 1

(b) MKT + LF1

Summary

- An important research problem: **Are the asset pricing models sparse**?
    - Schrödinger's Sparsity
- A new approach, the BayesIPCA Model, combines the **Bayesian framework of factor estimation** and the **chars-based model (IPCA)**.
    - An important extension for considering the **spike-and-slab prior** while estimating the conditional (latent) factor model.
- By avoiding pre-specified assumptions on sparsity or density, our approach **endogenously** determines whether the model is sparse or dense.



Summary

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  - An important extension for considering the **spike-and-slab prior** while estimating the conditional (latent) factor model.
- By avoiding pre-specified assumptions on sparsity or density, our approach **endogenously** determines whether the model is sparse or dense.
- Based on our method, we can:
  - Identify the global / separate sparsity levels of the asset-pricing model
  - Investigate the chars that drive alpha and betas
  - Resurrect the conditional CAPM