Estimation and Comparison of Beta-Pricing Models

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Background: Beta Pricing Models

- Beta-pricing models, such as CAPM, provide a foundational framework for explaining cross-sectional expected returns via factor risk premia and asset-factor exposures (betas).

$$E(\boldsymbol{r}_t) = \boldsymbol{B}\boldsymbol{\lambda}$$

- The two-pass regression is a standard estimation approach.
- Asset-specific time series regression for betas, factor exposures

$$r_{t,i} = a_i + \boldsymbol{\beta}_i \boldsymbol{f}_t + \boldsymbol{e}_{t,i}, \quad i = 1, ..., N$$

- Cross-sectional regression for lambdas, risk premia

$$\bar{r}_{t,i} = \hat{\boldsymbol{\beta}}_i \boldsymbol{\lambda} + \alpha_i, \quad i = 1, ..., N$$

Background: Factor Zoo

- Traded Factors:
  - excess market return
  - small-minus-big (size), high-minus-low (value) (e.g., Fama and French, 1993, JFE)
  - up-minus-down (momentum) (e.g., Jegadeesh and Titman, 1993, JF)
  - ...
- Nontraded Factors:
  - consumption growth (e.g., Breeden et al., 1989, JF)
  - market liquidity (e.g., Pastor and Stambaugh, 2003, JPE)
  - intermediary capital ratio(e.g., He et al., 2017, JFE)
  - ...

Motivation: Model Comparison

- Comparing Traded Factor Models:
  - GRS test (Sharpe Ratio Increase) (e.g., Gibbons, Ross, and Shanken, 1989, ECTA)
  - Bayesian Marginal Likelihood (Bayesian GRS-type comparison) (e.g., Barillas and Shanken,2018, JF, Chib, Zeng, and Zhao,2020, JF)
  - Only applies to traded factors
- Comparing Models with Nontraded Factors:
  - Two-pass Cross-Sectional R^2 (e.g., Kan, Robotti, and Shanken, 2013, JF)
    - CSR^2 tends to increase with more factors.
  - Hansen-Jaganathan Distance (e.g., Kan and Robotti, 2009, RFS)
    - HJD tends to decrease with more factors.

Motivation: Weak Factors

- Most of the nontraded factors are weakly correlated with test assets, which will cause the problem of reduced-rank of loading matrix $\boldsymbol{B}$.
- Inference on risk premia becomes invalid with weak factors. (e.g., Kan and Zhang,1999, JF; Kleibergen, 2009, JoE).
- F-rank statistics to test the identification of risk premia. (e.g., Kleibergen and Zhan,2020, JF)
- Giglio, Xiu, and Zhang (2025, JF) perform test assets selection and remove test assets exposed to weak factors

We provide a unified framework for simultaneous beta-pricing model comparison and risk premia estimation, which can exclude weak factors.

Our Solution: Bayesian Marginal Likelihood Comparison

- We develop a Bayesian framework for estimating beta-pricing models with traded and nontraded factors via marginal likelihoods.
- Simulations confirm that this criterion avoids the overfitting bias of traditional metrics such as cross-sectional R^2 and H-J distance.
- Empirically, the optimal model selects 8 traded factors, excluding all nontraded ones, and achieves excellent out-of-sample performance.

Simulation Evidence: Model Comparison

- True model: MKTRF+SMB+HML+HKMcapital
- All combinations of models:
  - 6 traded factors and 4 nontraded factors (1008 models)
  - Average across 100 simulations

Top 10 marginal likelihood models	logML	rank	CSR^2	H-J D
MKTRF+SMB+HML+HKMcapital	**55337.2**	**1.0**	62.6	0.0171
MKTRF+SMB+HML+CMA+HKMcapital	55284.7	3.7	63.4	0.0168
MKTRF+SMB+HML+RMW+HKMcapital	55284.5	3.8	63.3	0.0168
MKTRF+SMB+HML+UMD+HKMcapital	55284.1	3.9	63.5	0.0168
MKTRF+SMB+HML+PEAR+HKMcapital	55278.8	5.1	63.5	0.0168
MKTRF+SMB+HML+LIQ+HKMcapital	55278.8	5.1	63.6	0.0167
MKTRF+SMB+HML+PCEND+HKMcapital	55276.8	5.6	63.4	0.0169
MKTRF+SMB+HML	55243.6	11.5	48.0	0.0203
MKTRF+SMB+HML+RMW+CMA+HKMcapital	55231.2	13.0	64.1	0.0165
MKTRF+SMB+HML+CMA+UMD+HKMcapital	55231.1	13.1	64.2	0.0165

Empirical Highlight: Model Comarison

- 10 traded factors + 8 Nontraded factors:
  - MKTRF, SMB, HML, CMA, RMW, UMD, BAB, QMJ, IA, ROE
  - IndProd, Liq, LTY, M2_SA, PCEDG, PCEND, HKMcapital, PEAR

Rank	Top 5 Model	logML	CSR^2	H-J D	Prob
1	MKTRF + SMB + HML + RMW + CMA + UMD + BAB + QMJ	**322024**	49.03	0.0071	**1**
2	MKTRF + SMB + HML + RMW + CMA + UMD + BAB + QMJ + ROE	322008	49.68	0.0071	0
3	MKTRF + SMB + HML + RMW + CMA + UMD + BAB + QMJ + HKMcapital	321905	49.10	0.0071	0
4	MKTRF + SMB + HML + RMW + CMA + UMD + BAB + QMJ + ROE + HKMcapital	321895	49.74	0.0071	0
5	MKTRF + SMB + HML + RMW + CMA + UMD + BAB + QMJ + PCEDG	321891	49.10	0.0071	0

- Model averaging and model selection are equivalent when the top-1 model has a probability of almost 1.

Model

Model

- Assume that the distributions for traded (T) (e.g., Fama-French factors), nontraded factors (NT) (e.g., macroeconomic innovations), and returns are normal and take the stationary form

$$\begin{pmatrix} \boldsymbol{f}_t^T \\ \boldsymbol{f}_t^{NT} \\ \boldsymbol{r}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu}^T \\ \boldsymbol{\mu}^{NT} \\ \boldsymbol{\mu}_r \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_T & \boldsymbol{\Sigma}_{T,NT} & \boldsymbol{\Sigma}_{T,r} \\ \boldsymbol{\Sigma}_{NT,T} & \boldsymbol{\Sigma}_{NT} & \boldsymbol{\Sigma}_{NT,r} \\ \boldsymbol{\Sigma}_{r,T} & \boldsymbol{\Sigma}_{r,NT} & \boldsymbol{\Sigma}_r \end{pmatrix} \right)$$

- Now assume that these factors are in the SDF M_t, and suppose, following Hansen and Jagannathan (1997), that M_t is given by

$$M_t = 1 - \boldsymbol{\lambda}' \boldsymbol{\Sigma}_f^{-1} (\boldsymbol{f}_t - \boldsymbol{\mu}_f), \quad \boldsymbol{\lambda} = (\boldsymbol{\lambda}^T, \boldsymbol{\lambda}^{NT})$$

Model (contd)

- Under the no-arbitrage condition, we have the pricing restrictions

$$\mathbb{E}[M_t \boldsymbol{f}_t^{T'}] = 0, \quad \mathbb{E}[M_t \boldsymbol{r}_t'] = 0$$

- From the first of these pricing restrictions, one can show that

$$\boldsymbol{\mu}^T = \boldsymbol{\lambda}^T$$

and from the second that

$$\boldsymbol{\mu}_r = \boldsymbol{B}\boldsymbol{\lambda} = \boldsymbol{B}^T\boldsymbol{\lambda}^T + \boldsymbol{B}^{NT}\boldsymbol{\lambda}^{NT}$$

where $\boldsymbol{B} = \boldsymbol{\Sigma}_{f,r}\boldsymbol{\Sigma}_f^{-1}$ is the loading matrix.

Model (contd)

- Inserting these two pricing conditions together into the distribution of returns conditional on the factors, we get

$$\boldsymbol{r}_t = \boldsymbol{B}^{NT}\boldsymbol{\lambda}^{NT} + \boldsymbol{B}^{T}\boldsymbol{f}_t^{T} + \boldsymbol{B}^{NT}(\boldsymbol{f}_t^{NT} - \boldsymbol{\mu}^{NT}) + \boldsymbol{e}_t, \boldsymbol{e}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_e). \quad (1)$$

- With the distributional assumption of the factors,

$$\boldsymbol{f}_t = \boldsymbol{\mu} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_u) \quad (2)$$

  we get a restricted TS model that can be used to estimate all the parameters in *one pass* (Pastor and Stambaugh, 2003).

Likelihood of the Model

- The factor's mean is estimated, rather than demeaned ex ante

$$\boldsymbol{f}_t = \boldsymbol{\mu} + \boldsymbol{u}_t, \boldsymbol{u}_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}_u)$$

$$\boldsymbol{r}_t = \boldsymbol{B}^{NT}\boldsymbol{\lambda}^{NT} + \boldsymbol{B}^{T}\boldsymbol{f}_t^{T} + \boldsymbol{B}^{NT}(\boldsymbol{f}_t^{NT} - \boldsymbol{\mu}^{NT}) + \boldsymbol{e}_t, \boldsymbol{e}_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}_e)$$

- The likelihood of the model is given by

$$\mathcal{L} = P(\boldsymbol{R} \mid \boldsymbol{F}, \boldsymbol{\lambda}, \boldsymbol{B}, \Sigma_e, \boldsymbol{\mu})P(\boldsymbol{F} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}_u)$$
$$\propto -\frac{1}{2}\sum_{t=1}^{T} \boldsymbol{e}_t'\boldsymbol{\Sigma}_{\boldsymbol{e}}^{-1}\boldsymbol{e}_t - \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{u}_t'\boldsymbol{\Sigma}_{\boldsymbol{u}}^{-1}\boldsymbol{u}_t.$$

- Likelihood integrates cross-sectional pricing constraints and time-series variations.

Bayesian Marginal Likelihood

- Model comparison based on the likelihood?
  - Likelihood increases when adding more factors.
- The marginal likelihood integrates over parameters.

$$ML = \int \int P(\boldsymbol{R} \mid \boldsymbol{F}, \boldsymbol{\lambda}, \boldsymbol{B}, \Sigma_e, \boldsymbol{\mu}) P(\boldsymbol{F} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}_u) P(\boldsymbol{\lambda} \mid \boldsymbol{B}, \Sigma_e)$$
$$\times P(\boldsymbol{B} \mid \Sigma_e, \boldsymbol{\mu}) P(\Sigma_e \mid \boldsymbol{\mu}) P(\boldsymbol{\mu} \mid \boldsymbol{\Sigma}_u) P(\boldsymbol{\Sigma}_u) \, d\boldsymbol{\lambda} \, d\boldsymbol{B} \, d\Sigma_e \, d\boldsymbol{\mu} \, d\boldsymbol{\Sigma}_u$$

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$$\times P(\boldsymbol{B} \mid \Sigma_e, \boldsymbol{\mu}) P(\Sigma_e \mid \boldsymbol{\mu}) P(\boldsymbol{\mu} \mid \boldsymbol{\Sigma}_u) P(\boldsymbol{\Sigma}_u)\, d\boldsymbol{\lambda}\, d\boldsymbol{B}\, d\Sigma_e\, d\boldsymbol{\mu}\, d\boldsymbol{\Sigma}_u$$

- For weak factors, similar to the ridge penalty, increasing the dimension of $\boldsymbol{B}$ will decrease the $||\boldsymbol{R} - \boldsymbol{F}\boldsymbol{B}||^2$ but increase $||\boldsymbol{B}||^2$.
- The prior of $\boldsymbol{B}$ is a normal density proportional to $exp(-||\boldsymbol{B}||^2)$

$$ML = \int p(\theta) lik(data|\theta) d\theta \propto \int exp(-||\boldsymbol{R} - \boldsymbol{F}\boldsymbol{B}||^2 - ||\boldsymbol{B}||^2) d\theta$$

Bayesian Model Comparison for Beta-Pricing Model

- We have a full set of factors, which we define as $\boldsymbol{f}^*$. Different models choose different combinations within this full set
- The factor model is estimated based on this full set of factors.

$$\boldsymbol{f}_t^* = \boldsymbol{\mu} + \boldsymbol{u}_t, \quad \boldsymbol{f}_t \subset \boldsymbol{f}_t^*, \quad \boldsymbol{u}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_u)$$

- The return model is estimated based on the selected factors.

$$\boldsymbol{r}_t = \boldsymbol{B}^{NT}\boldsymbol{\lambda}^{NT} + \boldsymbol{B}^T\boldsymbol{f}_t^T + \boldsymbol{B}^{NT}(\boldsymbol{f}_t^{NT} - \boldsymbol{\mu}^{NT}) + \boldsymbol{e}_t$$

$$\boldsymbol{e}_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}_e)$$

- We must consider the full set of factors, $\boldsymbol{f}^*$, for model comparison, with the marginal likelihood comparable.

Prior Distribution*

We rely on the training sample prior distribution to make valid model comparisons (e.g., Chib and Zeng,2020, JBES, Chib et al.,2024 MS)

- The model-specific priors must be proper for valid model comparison.
- To ensure that differences in marginal likelihood reflect genuine model fit rather than variations in prior specifications, the prior distributions must be comparable across models.
- The chosen priors should be minimally subjective, requiring little user input while maintaining robustness in inference.

MCMC*

- The estimation is performed through the Gibbs Sampler.

Algorithm MCMC Sampling with Parameter Expressions

Initialize parameters: Set $\boldsymbol{\theta}^{(0)} = (\boldsymbol{B}^{(0)}, \boldsymbol{\Sigma}_e^{(0)}, \boldsymbol{\lambda}^{NT(0)}, \boldsymbol{\Sigma}_u^{(0)}, \boldsymbol{\mu}^{(0)})$

for $g = 1$ to $n_0 + M$ **do**

    Step 1: For $i = 1$ to $i = n$, sample $\boldsymbol{\beta}_i^{(g+1)} \mid \boldsymbol{\Sigma}_e^{(g)}, \boldsymbol{\lambda}^{NT(g)}, \boldsymbol{\Sigma}_u^{(g)}, \boldsymbol{\mu}^{(g)} \sim \mathcal{N}\left(\widehat{\boldsymbol{\beta}}_i^{(g)}, \mathcal{B}_i^{(g)}\right)$

    Step 2: For $i = 1$ to $i = n$, sample $\sigma_{ei}^2 \mid \boldsymbol{B}^{(g+1)}, \boldsymbol{\lambda}^{NT(g)}, \boldsymbol{\Sigma}_u^{(g)}, \boldsymbol{\mu}^{(g)} \sim \mathcal{IG}\left(\nu_{ei}^{(g)}, \delta_{ei}^{(g)}\right)$

    Step 3: Sample $\boldsymbol{\lambda}^{NT} \mid \boldsymbol{B}^{(g+1)}, \boldsymbol{\Sigma}_e^{(g+1)}, \boldsymbol{\Sigma}_u^{(g)}, \boldsymbol{\mu}^{(g)} \sim \mathcal{N}\left(\widehat{\boldsymbol{\lambda}}^{NT(g)}, \boldsymbol{L}^{NT(g)}\right)$

    Step 4: Sample $\boldsymbol{\mu}^{(g+1)} \mid \boldsymbol{B}^{(g+1)}, \boldsymbol{\Sigma}_e^{(g+1)}, \boldsymbol{\lambda}^{NT(g+1)}, \boldsymbol{\Sigma}_u^{(g)} \sim \mathcal{N}\left(\widehat{\boldsymbol{\mu}}^{(g)}, \boldsymbol{D}_1^{(g)}\right)$

    Step 5: Sample $\boldsymbol{\Sigma}_u^{(g+1)} \mid \boldsymbol{B}^{(g+1)}, \boldsymbol{\Sigma}_e^{(g+1)}, \boldsymbol{\lambda}^{NT(g+1)}, \boldsymbol{\mu}^{(g+1)} \sim \mathcal{IW}\left(\nu_u^{(g)}, \boldsymbol{S}_u^{(g)}\right)$

end for

After burn-in, use samples $\{\boldsymbol{\theta}^{(g)}\}_{g=1}^{N}$ to estimate the posterior distribution.

- Output from this sampling is used to estimate the marginal likelihood by Chib (1995)'s method.

Simulation

Data

- Simulation uses parameters calibrated from empirical data.
- Jan 1985 to Dec 2023.
- A large cross-section of test assets, including 302 equity portfolios downloaded from French's website.
- 10 Traded factors:
  - MKTRF, SMB, HML, CMA, RMW, UMD, BAB, QMJ, IA, ROE
- 8 Nontraded factors:
  - IndProd, Liq, LTY, M2_SA, PCEDG, PCEND, HKMcapital, PEAR

Simulation for Weak (Near Zero Loading) Factors

- Data is simulated from

$$\boldsymbol{f}_t = \boldsymbol{\mu} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_u)$$

$$\boldsymbol{r}_t = w\boldsymbol{B}^{NT}\boldsymbol{\lambda}^{NT} + \boldsymbol{B}^{T}\boldsymbol{f}_t^{T} + w\boldsymbol{B}^{NT}(\boldsymbol{f}_t^{NT} - \boldsymbol{\mu}^{NT}) + \boldsymbol{e}_t, \quad \boldsymbol{e}_t \sim \mathcal{N}(0, \boldsymbol{\Sigma}_e)$$

  where $\boldsymbol{f}^T$ includes FF5 + UMD as traded factor benchmark, and the nontraded factor is evaluated individually.

- w is the weakness level. Smaller w means weaker.

- log BF (Bayes Factor): difference in log Marginal Likelihood between the models with and without the nontraded factor.

Simulation for Weak (Near Zero Loading) Factors

- Even if $\lambda = 0$, the strong factor ($w = 10$) is selected because it helps explain the time-series variation.
- Although $\lambda = 0.01$, the weak factor ($w = 0.1$) is dropped because the overall contribution does not exceed the penalty.

	log *BF*	log *BF*	log *BF*	log *BF*	log *BF*	log *BF*	log *BF*	log *BF*
	IndProd	LIQ	LTY	M2	PCEDG	PCEND	HKMcapital	PEAR
			Panel A: $\lambda = 0$					
w=10	10966	8787	10493	8052	5594	12329	14315	5045
w=1	103	34	80	20	-37	146	182	-49
w=0.1	-121	-121	-122	-123	-125	-122	-118	-123
			Panel B: $\lambda = 0.01$					
w=10	16763	8874	10654	21735	6055	17452	14005	5236
w=1	311	36	86	555	-33	344	180	-43
w=0.1	-121	-123	-123	-120	-121	-120	-120	-123

Simulation for Constant-loading (level) Factors*

- Another type of weak factor with constant loading: $\boldsymbol{B}$ does not have sufficient cross-sectional variation.

- Problems arise when there is more than one constant-loading factor.

- Same loading $\boldsymbol{B}_{\text{CLF}_1} = \boldsymbol{B}_{\text{CLF}_2} = 0.1$ for all test assets.

$CLF_1 + CLF_2$	No CLF	CLF_1	CLF_2
Panel A: $\lambda = 0$			
120566	118824	118838	**120641**
Panel B: $\lambda = 0.01$			
120523	118144	118740	**120595**

Empirical

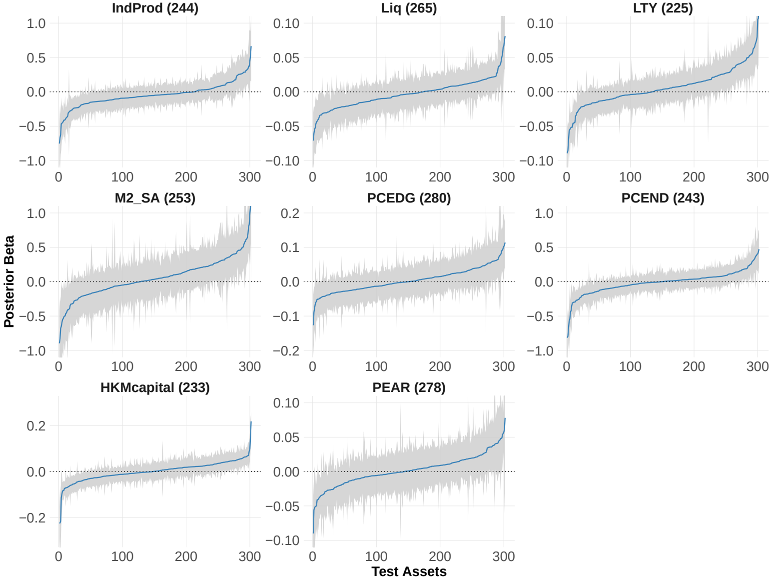
Absolute Test of Single Nontraded Factor

- Third row: log *BF* (Bayes Factor)

	IndProd	LIQ	LTY	M2_SA	PCEDG	PCEND	HKMcapital	PEAR
	-0.68	4.57	1.36	-0.38	0.82	-0.70	1.78	-3.68
CAPM	[-0.83,-0.53]	[3.56,5.56]	[0.74,1.96]	[-0.45,-0.32]	[0.50,1.13]	[-0.84,-0.56]	[1.13,2.36]	[-4.24,-3.09]
	-424	-308	405	-312	340	-300	1291	-22
	0.07	-1.43	0.46	0.16	-1.10	-0.07	0.67	0.40
FF5	[-0.06,0.20]	[-2.31,-0.52]	[-0.43,1.34]	[0.09,0.24]	[-1.63,-0.55]	[-0.21,0.07]	[-0.03,1.35]	[-1.05,1.69]
	-332	-251	-351	-427	-122	-184	1	-239
	-0.04	-2.48	2.98	0.15	1.31	-0.25	1.50	2.52
All	[-0.17,0.09]	[-3.51,-1.45]	[2.15,3.75]	[0.08,0.22]	[0.45,1.99]	[-0.40,-0.11]	[0.76,2.24]	[1.55,3.40]
	-257	-261	-174	-350	-130	-162	-111	-182

- LTY and PCEDG show marginal contribution over CAPM, and HKMcapital even improves over FF5.
- No nontraded factor enhances the All benchmark.

Posterior Beta of Nontraded Factors (mostly zero)



Out-of-sample Model Performance*

- Estimate model implied tangency portfolio weights (1985-2003).
- Fix weights for out-of-sample evaluation (2004-2023).

Panel A : Model-implied

	Bayes SR	WLS SR	Bayes MDD	WLS MDD
Rank 1	0.983	0.600	0.231	0.428
Rank 2	0.975	0.613	0.246	0.407
Rank 3	0.878	0.588	0.307	0.445
CAPM	0.568	0.568	0.482	0.482
FF3	0.650	0.559	0.428	0.437
FF5	0.883	0.796	0.256	0.246
ALL	0.749	0.813	0.560	0.384

Panel B: EW and MVE

EW SR	MVE SR		EW MDD	MVE MDD
0.526	0.566		0.491	0.472

Summary

Summary

- Our Bayesian framework is designed to enable beta-pricing model estimation and comparisons via marginal likelihoods.
- Empirically, we evaluate each nontraded factor for its incremental contribution and find all tested nontraded factors to be weak.
- Our framework selects models that satisfy cross-sectional pricing constraints, capture time-series dynamics, and exclude weak factors.

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